CET – MATHEMATICS – 2011

VERSION CODE: A – 4

1. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, then the value of $x^{b+c} \cdot y^{c+a} \cdot z^{a+b}$ is ....

   1) -1 2) 0 3) 2 4) 1

Ans: (4)

$\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k \Rightarrow x = e^{k\,(b-c)},\ y = e^{k\,(c-a)},\ z = e^{k\,(a-b)}$

$x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = e^{k(b^2-c^2)} \cdot e^{k(c^2-a^2)} \cdot e^{k(a^2-b^2)} = e^0 = 1$

2. The sum of the first n terms of $\frac{1^2}{1} + \frac{1^2+2^2}{1+2} + \frac{1^2+2^2+3^2}{1+2+3} + \ldots\ldots$ is ....

   1) $\frac{2n^2-n}{3}$ 2) $\frac{n(n+2)}{3}$ 3) $\frac{2n^2+n}{3}$ 4) $\frac{n^2-2n}{3}$

Ans: (2)

Put n = 2 $\frac{1^2}{1} + \frac{1^2+2^2}{1+2} = 1 + \frac{5}{3} = \frac{8}{3}$

Check with options, option (2) only satisfies

3. If n is an odd positive integer and $(1 + x + x^2 + x^3)^n = \sum_{r=0}^{3n} a_r x^r$, then

   $a_0 - a_1 + a_2 - a_3 + \ldots - a_{3n}$ is **(print mistake)**

   1) 0 2) -1 3) 1 4) 4^n

Ans: (1)

Put x = - 1

$(1 - 1 + 1 - 1)^n = \sum_{r=0}^{3n} a_r x^r \Rightarrow 0 = a_0 - a_1 + a_2 - a_3 + \ldots\ldots$

4. If r^{th} and $(r + 1)^{th}$ terms in the expansion of $(p + q)^n$ are equal, then $\frac{(n+1)q}{r(p+q)}$ is

   1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) 1 4) 0

Ans: (3)

$T_r = {}^nc_{r-1}\, p^{n-r+1} \,.\, q^{r-1} = (T_{r+1}) = {}^nc_r\, p^{n-r}.q^r$ (given)

$\Rightarrow \dfrac{n!}{(n-r+1)!(r-1)!}\, p^{n-r+1} \,.\, q^{r-1} = \dfrac{n!}{(n-1)!r!}\, p^{n-r} \,.\, q^r$

$\dfrac{p}{(n-r+1)} = \dfrac{q}{r} \Rightarrow pr = nq - rq + q$

$(p + q)\, r = q\,(n + 1)$

$\dfrac{(n+1)q}{(p+q)r} = 1$

5. If α, β and γ are roots of $x^3 - 2x + 1 = 0$, then the value of $\sum\left(\dfrac{1}{\alpha+\beta-\gamma}\right)$ is

1) $\frac{1}{2}$ 2) 0 3) -1 4) $\frac{-1}{2}$

Ans: (3)

$x^3 - x - x + 1 = 0\ x^2\,(x^2 - 1) - (x - 1) = 0$

$x = 1,\ x^2 + x - 1 = 0\ \alpha = 1,\ \beta = \dfrac{-1+\sqrt{5}}{2},\ \gamma = \dfrac{-1-\sqrt{5}}{2}$

$\dfrac{1}{\alpha+\beta-\gamma} = \dfrac{1}{1+\sqrt{5}}\quad \dfrac{1}{\beta+\gamma-\alpha} = \dfrac{1}{-2}\quad \dfrac{1}{\gamma+\alpha-\beta} = \dfrac{1}{1-\sqrt{5}}$

$\sum \dfrac{1}{\alpha+\beta+\gamma} = -\dfrac{1}{2} - \dfrac{1}{2} = -1$

6. Define a relation R on A = {1, 2, 3, 4} as $_xR_y$ if x divides y. R is

1) equivalence 2) symmetric and transitive

3) reflexive and symmetric 4) reflexive and transitive

Ans: (4)

Reflexive and transitive

7. The negation of $p \rightarrow (\sim p \vee q)$ is

1) $p \wedge \sim q$ 2) $p \rightarrow q$ 3) $p \rightarrow \sim(p \vee q)$ 4) $p \vee (p \vee \sim q)$

Ans: (1)

$\sim (p \rightarrow (\sim p \vee q)$ is $p \wedge \sim (\sim p \vee a)$

$\equiv p \wedge (p \wedge \sim q) \equiv (p \wedge \sim q)$

8. In any triangle ABC, the simplified form of $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$ is ……..

1) $a^2 + b^2$ 2) $\frac{1}{a^2} - \frac{1}{b^2}$ 3) $\frac{1}{a^2 - b^2}$ d) $a^2 - b^2$

Ans: (2)

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1-2\sin^2 A}{a^2} - \frac{1-2\sin^2 B}{b^2} = \frac{1}{a^2} - \frac{1}{2R^2} - \frac{1}{b^2} + \frac{1}{2R^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

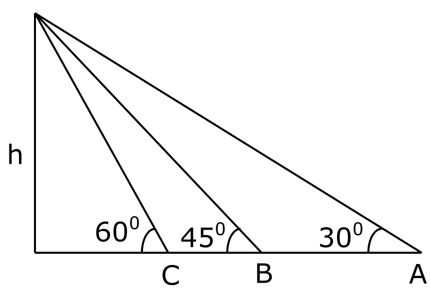
9. Angles of elevation of the top of a tower from three points (collinear) A, B and C on a road leading to the foot of the tower are 30^o, 45^o and 60^o respectively. The ratio of AB to BC is

1) 2: $\sqrt{3}$ 2) 1: 2 3) $\sqrt{3}$: 2 4) $\sqrt{3}$: 1

Ans: (4)

AB = h (cot 30^0 – cot 45^0) = h ($\sqrt{3}$ - 1)

BC = h (cot 45^0 – cot 60^0) = h (1 - $\frac{1}{\sqrt{3}}$)



$$\frac{AB}{BC} = \frac{\sqrt{3}-1}{1-\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$\sqrt{3}$: 1

10. The value of sin 10 . sin 30 . sin 70 is **(unit is not given)**

1) $\frac{1}{16}$ 2) $\frac{\sqrt{3}}{16}$ 3) $\frac{3}{16}$ 4) $\frac{1}{8}$

Ans: (1)

sin 10^o sin 30^o sin 50^o sin 70^o

$$\sin 10^0 \left(\frac{1}{2}\right) \frac{1}{2} \{\cos 20^0 - \cos 120^0\} = \frac{1}{4} \sin 10^0 (\cos 20^0 + \frac{1}{2})$$

$$\frac{1}{4} \sin 10^0 \cos 20^0 - \frac{1}{8} \sin 10^0 = \frac{1}{4} \times \frac{1}{2} (\sin 30^0 - \sin 10^0) + \frac{1}{8} \sin 10^0 = \frac{1}{16}$$

11. Locus of a point which moves such that its distance from the X-axis is twice its distance from the line x – y = 0 is

1) $x^2 - 4xy - y^2 = 0$ 2) $x^2 - 4xy + y^2 = 0$

3) $2x^2 - 4xy + y^2 = 0$ 4) $x^2 + 4xy - y^2 = 0$

Ans : (3)

Let P (x, y) be a point on locus cond $|y| = 2\left|\frac{x-y}{\sqrt{2}}\right| \Rightarrow |y| = \sqrt{2}\ |x - y|$

squaring $y^2 = 2(x^2 + y^2 - 2xy)$

$2x^2 + y^2 - 4xy = 0$

12. The points A (1, 2), B (2, 4) and C (4, 8) form a/an …

1) right angled triangle	2) straight line

3) equilateral triangle	4) isosceles triangle

Ans: (2)

$\left.\begin{matrix}\text{slope of } AB = 2\\ \text{slope of } BC = 2\end{matrix}\right\} \Rightarrow$ A, B, C represent straight line

13. If lines represented by $x + 3y - 6 = 0$, $2x + y - 4 = 0$ and $kx - 3y + 1 = 0$ are concurrent, then the value of k is ………

1) $\frac{-6}{19}$	2) $\frac{-19}{6}$	c) $\frac{19}{6}$	4) $\frac{6}{19}$

Ans: (3)

Solving $x + 3y = 6$

$2x + y = 4$

$\Rightarrow (x, y) = \left(\frac{6}{5}, \frac{8}{5}\right)$

Sub in $kx - 3y + 1 = 0$

$\frac{6k}{5} - \frac{24}{5} + 1 = 0 \Rightarrow k = \frac{19}{6}$

14. $\lim_{x \to a}\left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - \sqrt[2]{x}}\right]$ = …………… **(Question incorrect)**

1) $\frac{2}{3\sqrt{3}}$	2) $\frac{3\sqrt{3}}{2}$	3) $\frac{2}{\sqrt{3}}$	4) $\frac{2}{3}$

Ans:

Direct substitution of x = a, answer is 0, which is not given in the option

15. If $f(x) = \begin{cases} \frac{\log x}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$ is continuous at x = 1, then the value of k is ………

1) e	2) 1	3) -1	4) 0

Ans: (2)

$$k = \lim_{x \to 1} \frac{\log x}{x-1} = \lim_{x \to 1} \frac{\frac{1}{x}}{1}$$ [using L'Hospital Rule]

16. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then A . A' is ……….

1) A^2 2) –A 3) A 4) I

Ans: (4)

AA' = I [Standard Result]

17. If $\begin{bmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{bmatrix}$ is singular, then the value of x is ………

1) 0 2) 1 3) 3 4) 2

Ans: (4)

Expanding: $x - 2 - 1 - 2(1 - x) - 1(1 - x^2 + 2x) = 0$

$x - 3 - 2 + 2x - 1 + x^2 - 2x = 0$

$x^2 + x - 6 = 0$

x = 2 satisfies the above equation

18. If A and B are symmetric matrices of the same order, then which one of the following is NOT true?

1) AB – BA is symmetric 2) AB + BA is symmetric

3) A – B is symmetric 4) A + B is symmetric

Ans: (1)

AB – BA is skew symmetric

19. If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix} 1 & \omega^2 & 1-\omega^4 \\ \omega & 1 & 1+\omega^5 \\ 1 & \omega & \omega^2 \end{vmatrix}$ is ……

1) 4 2) ω^2 3) $\omega^2 - 4$ 4) -4

Ans: (3)

$1\,[\omega^2 - \omega - 1] - \omega^2\,[1 - 1 - \omega^2] + (1 - \omega)\,[\omega^2 - 1]$

$= \omega^2 - \omega - 1 + \omega + \omega^2 - 1 - 1 + \omega$

$= \omega^2 - 3 + \omega + \omega^2 = \omega^2 - 4$

20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then angle between $\vec{a}$ & $\vec{b}$ is …..

1) π 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: (2)

$|\vec{a} + \vec{b}|^2 = |\vec{c}|^2$

$\Rightarrow |\vec{a}| + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$

$\Rightarrow \vec{a} \cdot \vec{b} = -\frac{1}{2} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^{o}$

21. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are noncoplanar, then the value of $\vec{a} \cdot \left\{\frac{\vec{b} \times \vec{c}}{3\vec{b}.(\vec{c} \times \vec{a})}\right\} - \vec{b} \cdot \left\{\frac{\vec{c} \times \vec{a}}{3\vec{c}.(\vec{a} \times \vec{b})}\right\}$ is

1) $\frac{1}{6}$ 2) $\frac{-1}{6}$ 3) $\frac{-1}{3}$ 4) $\frac{-1}{2}$

Ans: (2)

$$3\left(\frac{\vec{a}.(\vec{b} \times \vec{c})}{\vec{a}.(\vec{b} \times \vec{c})}\right) - \frac{1}{2}\left(\frac{\vec{b}.(\vec{c} \times \vec{a})}{\vec{b}.(\vec{c} \times \vec{a})}\right) = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

22. If $2\hat{i} + 3\hat{j}$, $\hat{i} + \hat{j} + \hat{k}$ and $\lambda\hat{i} + 4\hat{j} + 2\hat{k}$ taken in an order are conterminous edges of a parallelepiped of volume 2 Cu units, then value of λ is ……

1) 4 2) 3 3) 2 4) -4

Ans: (1)

$$\begin{vmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ \lambda & 4 & 2 \end{vmatrix} = 2$$

$2(2 - 4) - 3(2 - \lambda) = 2$

$-4 - 6 + 3\lambda = 2 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$

23. A unit vector perpendicular to both $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$ is ………..

1) $\frac{3\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ 2) $2\hat{i} + \hat{j} + \hat{k}$ 3) $\frac{(2\hat{i} - \hat{j} - \hat{k})}{\sqrt{6}}$ 4) $(2\hat{i} - \hat{j} + \hat{k})\sqrt{6}$

Ans: (3)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\hat{n} = \frac{(2\hat{i} - \hat{j} - \hat{k})}{\sqrt{6}}$$

24. The digit in the unit's place of 7^{171} + (177)! is

1) 0 2) 1 3) 1 4) 3

Ans: (4)

$7^2 \equiv -1 \pmod{10}$

$(7^2)^{85} \equiv -1 \pmod{10}$

$7^{170} . 7 \equiv -7 \pmod{10} \equiv 3 \pmod{10}$

unit digit = 3

but unit digit in (177)! = 0

25. The sum of all positive divisors of 242 except 1 and itself is ………..

1) 399 2) 342 3) 242 4) 156

Ans: (4)

2 | 242
11 | 121
11

$242 = 11^2 . 2^1$

$$S(242) = \frac{11^3 - 1}{10} . \frac{2^2 - 1}{1} = (133)(3) = 399$$

Req. sum = 399 – (242 + 1) = 156

26. On the set of all nonzero reals, an operation $*$ is defined as $a * b = \frac{3ab}{2}$. In this group, a solution of $(2 * x) * 3^{-1} = 4^{-1}$ is ……….

1) $\frac{3}{2}$ 2) $\frac{1}{6}$ 3) 1 4) 6

Ans: (2)

$$4^{-1} = \frac{\left(\frac{4}{9}\right)}{4} = \frac{1}{9}$$

$$GE \Rightarrow 2 * x = 4^{-1} * 3 \Rightarrow \frac{3.2x}{2} = \frac{1}{9} * 3 \Rightarrow 3x = \frac{1}{2} \Rightarrow x = \frac{1}{6}$$

27. G = $\left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix}, \text{ x is a nonzero real number}\right\}$ is a group with respect to matrix multiplication. In this group, the inverse of $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ is ……..

1) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ 3) $\begin{bmatrix} 3/4 & 3/4 \\ 3/4 & 3/4 \end{bmatrix}$ 4) $\begin{bmatrix} 4/3 & 4/3 \\ 4/3 & 4/3 \end{bmatrix}$

Ans: (3)

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \therefore \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} \end{pmatrix}$$

28. If $2x^2 + 2y^2 + 4x + 5y + 1 = 0$ and $3x^2 + 3y^2 + 6x - 7y + 3k = 0$ are orthogonal, then value of k is

1) $\frac{-17}{12}$ 2) $\frac{-12}{17}$ 3) $\frac{12}{17}$ 4) $\frac{17}{12}$

Ans: (1)

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2 \Rightarrow 2\,(1) + \frac{5}{2}\left(-\frac{7}{6}\right) = \frac{1}{2} + k \Rightarrow k = -\frac{17}{12}$$

29. The total number of common tangents of $x^2 + y^2 - 6x - 8y + 9 = 0$ and $x^2 + y^2 = 1$ is ……..

1) 1 2) 3 3) 2 4) 4

Ans: (2)

$c_1 = (3, 4)$ $c_2 = (0, 0)$

$r_1 = 4$ $r_2 = 1$

$c_1\, c_2 = 5$ $r_1 + r_2 = 5$

circles touch externally

$\therefore$ No of common tangents 3

30. The center of a circle which cuts $x^2 + y^2 + 6x - 1 = 0$, $x^2 + y^2 - 3y + 2 = 0$ and $x^2 + y^2 + x + y - 3 = 0$ orthogonally is …………

1) $\left(\frac{-1}{7}, \frac{9}{7}\right)$ 2) $\left(\frac{1}{7}, \frac{-9}{7}\right)$ 3) $\left(\frac{-1}{7}, \frac{-9}{7}\right)$ 4) $\left(\frac{1}{7}, \frac{9}{7}\right)$

Ans: (1)

Req. point = radical center

$S_1 - S_2 = 0 \Rightarrow 6x + 3y - 3 = 0$

$S_2 - S_3 = 0 \Rightarrow - x - 4y + 5 = 0$

$\Rightarrow - 6x - 24y + 30 = 0$

$\Rightarrow - 21y + 27 = 0$

$y = \frac{27}{21} = \frac{9}{7}$

$\therefore x = 5 - 4\left(\frac{9}{7}\right)$

$= \frac{-1}{7}$

$\Rightarrow (x, y) = \left(-\frac{1}{7}, \frac{9}{7}\right)$

31. The length of the latus rectum of $3x^2 - 4y + 6x - 3 = 0$ is

1) 3 2) 2 3) $\frac{4}{3}$ 4) $\frac{3}{4}$

Ans: (3)

$3x^2 + 6x = 4y + 3 \Rightarrow 3(x + 1)^2 = 4y + 6 \Rightarrow (x + 1)^2 = \frac{4}{3}\left(y + \frac{6}{4}\right)$

LLR = $\frac{4}{3}$

32. The sum of the reciprocals of focal distances of a focal chord PQ of $y^2 = 4ax$ is

1) $\frac{1}{2a}$ 2) 2a 3) a 4) $\frac{1}{a}$

Ans: (4)

consider latus rectum, with ends (a, 2a), (a, -2a).

Sum of reciprocals of focal distances is $\frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}$

33. If the foci of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ coincide, then value of a is **(question incorrect)**

1) 1 2) 2 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

Ans: (answer not found)

Foci of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ is $(\pm\sqrt{12}, 0)$

Foci of $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ is $(\pm\sqrt{a^2 + 3}, 0)$

Given $a^2 + 3 = 12 \Rightarrow a^2 = 9 \Rightarrow a = 3$

34. The equation of a hyperbola whose asymptotes are $3x \pm 5y = 0$ and vertices are $(\pm 5, 0)$ is

1) $9x^2 - 25y^2 = 225$ 2) $25x^2 - 9y^2 = 225$

3) $5x^2 - 3y^2 = 225$ 4) $3x^2 - 5y^2 = 25$

Ans: (1)

The asymptotes are $\frac{x}{5} + \frac{y}{3} = 0$ and $\frac{x}{5} - \frac{y}{3} = 0$

The equation of hyperbola is $\frac{x^2}{5^2} - \frac{y^2}{9} = 1 \Rightarrow 9x^2 - 25y^2 = 225$

OR

Only option (1) has its vertices as $(\pm 5, 0)$

35. The domain of $f(x) = \sin^{-1}\left[\text{Log}_2\left(\frac{x}{2}\right)\right]$ is

1) $4 \le x \le 6$ 2) $1 \le x \le 4$ 3) $0 \le x \le 4$ 4) $0 \le x \le 1$

Ans: (2)

$f(x) = \sin^{-1}\left[\log_2\left(\frac{x}{2}\right)\right]$

$-1 \le \log_2 \frac{x}{2} \le 1$

$\log_2 \frac{x}{2} \le 1 \Rightarrow \frac{x}{2} \le 2$

$x \le 4$

$\Rightarrow 1 \le x \le 4$

$\log_2 \frac{x}{2} \ge -1$

$\frac{x}{2} \ge 2^{-1}$

$\frac{x}{2} \ge \frac{1}{2}$

$x \ge 1$

Also, $\frac{x}{2} > 0$

$x > 0$

$x \ge 1$

36. If $\text{Tan}^{-1} x = \frac{\pi}{4} - \text{Tan}^{-1}\left(\frac{1}{3}\right)$, then x is

1) $\frac{1}{6}$ 2) $\frac{1}{4}$ 3) $\frac{1}{2}$ 4) $\frac{1}{3}$

Ans: (3)

$\text{Tan}^{-1} x + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

Clearly, $x = \frac{3-1}{3+1} = \frac{1}{2}$

37. A value of θ satisfying Sin 5θ - Sin 3θ + Sin θ = 0 such that $0 < \theta < \frac{\pi}{2}$ is

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{12}$

Ans: (3)

$$\text{Sin}\left(\frac{5\pi}{6}\right) - \text{Sin}\ \frac{\pi}{2} + \text{Sin}\ \frac{\pi}{6} = \frac{1}{2} - 1 + \frac{1}{2} = 0$$

38. The value of $\left|\frac{1+i\sqrt{3}}{\left(1+\frac{1}{i+1}\right)^2}\right|$ is

1) $\frac{4}{5}$ 2) $\frac{5}{4}$ 3) 9 4) 20

Ans: (1)

$$\left|\frac{1+i\sqrt{3}}{\left(1+\frac{1}{\lambda+1}\right)^2}\right| = \left|\frac{1+i\sqrt{3}}{\frac{(i+2)^2}{(i+1)^2}}\right| = \frac{\sqrt{1+3}}{1+4}\ \text{x}\ (1+1) = \frac{2\,\text{x}\,2}{5}$$

39. If ω is an imaginary cube root of unity, then the value of

$(1 - \omega + \omega^2)\ (1 - \omega^2 + \omega^4)\ (1 - \omega^4 + \omega^8)$ …. (2n factors) is

1) 0 2) 1 3) 2^n 4) 2^{2n}

Ans: (4)

Put n = 1

GE = $(1 - \omega + \omega^2)\ (1 - \omega^2 + \omega)$

= $(-2\omega)\ (-2\omega^2)$

= $4\omega^3 = 4$

40. If P (x, y) denotes z = x + iy in Argand's plane and $\left|\frac{z-1}{z+2i}\right|$=1, then the locus of P is a/an

1) straight line 2) circle 3) ellipse 4) hyperbola

Ans: (1)

$|z - 1|^2 = |z + 2i|^2$

$(x + iy - 1)\ (x - iy - 1) = (x + iy + 2i)$

41. If $\sqrt{r} = ae^{\theta\cot\alpha}$ where a and α are real numbers, then $\frac{d^2r}{d\theta^2}$ - 4 r $\cot^2\alpha$ is

1) 0 2) 1 3) $\frac{1}{r}$ 4) r

Ans: (1)

$$\frac{1}{2}\log r = \log \alpha + \theta \cot \alpha$$

$$\frac{1}{2} \cdot \frac{1}{r} \frac{dr}{d\theta} = 0 + \cot \alpha$$

$$\frac{dr}{d\theta} = 2r \cot \alpha$$

$$\frac{d^2r}{d\theta^2} = 2 \cot \alpha \frac{dr}{d\theta} = 2 \cot \alpha \cdot 2 r \cot \alpha$$

$$\frac{d^2r}{d\theta^2} = 4 r \cot^2 \alpha = 0$$

42. The derivative of $\tan^{-1}\left[\frac{\sin x}{1+\cos x}\right]$ with respect to $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]$ is

1) -2 2) 0 3) -1 4) 2

Ans: (3)

$$u = \tan^{-1} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$v = \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$\frac{du}{dv} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

43. $\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ is

1) $\frac{-3}{4}$ 2) $\frac{-1}{2}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

Ans: (4)

$$x = 2\cos\theta$$

$$\frac{2+2\cos\theta}{2-2\cos\theta} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \cot^2\frac{\theta}{2}$$

$y = \cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2} = \frac{1+\frac{x}{2}}{2} = \frac{1}{2} + \frac{x}{2}$

$\frac{dy}{dx} = \frac{1}{4}$

44. If $f(x) = \frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x}$, then $f^{|}\left(\frac{\pi}{4}\right)$ is

1) $-\sqrt{3}$ 2) 0 3) $\frac{1}{\sqrt{3}}$ 4) $\sqrt{3}$

Ans: (2)

$f(x) = \frac{\sin^3 x}{\sin x + \cos x} + \frac{\cos^3 x}{\cos x + \sin x}$

$f(x) = \sin^2 x - \sin x \cos x + \cos^2 x$

$= 1 - \sin x \cos x = 1 - \frac{\sin 2x}{2}$

$f'(x) = 0 - \frac{1}{2} \cdot 2\cos 2x = -\cos 2x$

$f'\left(\frac{\pi}{4}\right) = 0$

45. If $\cos^{-1}\left(\frac{y}{b}\right) = n \operatorname{Log}\left(\frac{x}{n}\right)$, then

1) $xy_1 - \sqrt{b^2 - y^2} = 0$ 2) $y_1 = x\sqrt{b^2 - y^2}$

3) $xy_1 + n\sqrt{b^2 - y^2} = 0$ 4) $xy_1 = n\sqrt{b^2 - y^2}$

Ans: (3)

$\cos^{-1}\left(\frac{y}{b}\right) = 2\log\left(\frac{x}{2}\right)$

diff. w.r.t x $= -\frac{1}{\sqrt{1-\frac{y^2}{b^2}}} \cdot \frac{1}{b} \cdot y_1 = 2 \cdot \frac{1}{\frac{x}{2}} \cdot \frac{1}{2}$

$= -\frac{1}{\sqrt{b^2 - y^2}} \cdot y_1 = \frac{2}{x} \Rightarrow -xy_1 = 2\sqrt{b^2 - y^2}$

$xy_1 + 2\sqrt{b^2 - y^2} = 0$

46. Area of a triangle formed by tangent and normal to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 at P $\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$ with x-axis is

1) $\frac{b(a^2+b^2)}{4a}$ 2) $\frac{ab\sqrt{a^2-b^2}}{4}$ 3) $\frac{ab\sqrt{a^2+b^2}}{4}$ 4) 4ab

Ans: (1)

Put a = b

$\frac{xx_1}{a^2}$ + $\frac{yy_1}{a^2}$ = 1

$\frac{x}{\sqrt{2}a}$ + $\frac{y}{\sqrt{2}a}$ = 1

area = $\frac{1}{2}$ $\sqrt{2}$ a x $\frac{a}{\sqrt{2}}$ = $\frac{a^2}{2}$

$\therefore$ option (2), (3) and (4) are incorrect

hence option (1) is correct

47. The angle between y^2 = 4x and x^2 + y^2 = 12 at a point of their intersection is

1) $\tan^{-1}\left(\frac{1}{2}\right)$ 2) $\tan^{-1}2\sqrt{2}$ 3) $\tan^{-1}2$ 4) $\tan^{-1}\sqrt{2}$

Ans: (2)

$2yy_1 = 4$ $2x + 2yy_1 = 0$

$y_1 = \frac{4}{2y}$ $yy_1 = -x$

$y_1 = \frac{2}{y}$ $y_1 = -\frac{x}{y}$

$m_1 = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$ $m_2 = -\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{1+\left(-\frac{1}{2}\right)}\right| = \left(\frac{\sqrt{2}}{\frac{1}{2}}\right) . 2\sqrt{2}$$

48. A sphere increases its volume at the rate of π cc/s. The rate at which its surface area increases when the radius is 1 cm is

1) $\frac{\pi}{2}$ sq. cm/s 2) $\frac{3\pi}{2}$ sq. cm/s 3) π sq. cm/s 4) 2π sq. cm/s

Ans: (4)

$V = \frac{4}{3}\pi r^3$ $\qquad$ $S = 4\pi r^2$

$\frac{dV}{dt} = 4\pi \cdot r^2 \frac{dr}{dt}$ $\qquad$ $\frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt}$

$\pi = 4\pi \frac{dr}{dt}$ $\qquad$ $= s^2\pi \cdot 1 \cdot \frac{1}{4} = 2\pi$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4}$

49. The value of $\int_0^4 |x-1|\, dx$ is

1) 1 $\qquad$ 2) 4 $\qquad$ 3) 5 $\qquad$ 4) $\frac{5}{2}$

Ans: (3)

$\int_0^4 |x-1|\ dx = \int_0^1 -(x-1)\ dx + \int_1^4 (x-1)\,dx$

$= + \int_0^1 (1-x)\ dx + \int_1^4 (x-1)\,dx$

$= x - \frac{x^2}{2}\Big|_0^1 + \frac{x^2}{2} - x\Big|_1^4$

$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 = 5$

50. If $I_n = \int_0^4 \tan^n x\,dx$, where n is a positive integer, then $I_{10} + I_8$ is

1) 9 $\qquad$ 2) $\frac{1}{7}$ $\qquad$ 3) $\frac{1}{8}$ $\qquad$ 4) $\frac{1}{9}$

Ans: (4)

$I_n = \int_0^{\pi/4} \tan^n x\ dx = \int_0^{\pi/4} \tan^{n-2} x\ .\ \tan^2 x\ dx = \int_0^{\pi/4} \tan^{n-2}(x)\ [\sec^2 x - 1]\ dx$

$I_n = \int_0^{\pi/4} \tan^{n-2}(x)\ dx - I_{n-2}$

$I_n + I_{n-2} = \frac{\tan^{(n-1)}(x)}{n-1}\Big|_0^{\pi/4}$

$I_n + I_{n-2} = \frac{1}{n-1}$ $\qquad$ $\therefore I_{10} + I_8 = \frac{1}{9}$

51. $\int e^x \left[\frac{\sin x + \cos x}{1 - \sin^2 x}\right] dx$ is

1) $e^x \tan x + c$ 2) $(e^x. \sec x) + c$

3) $e^x \cot x + c$ 4) $(e^x \operatorname{cosec} x) + c$

Ans: (2)

$\int e^x \left(\frac{\sin x + \cos x}{1 - \sin^2 x}\right) dx$

$= \int e^x (\tan x \,.\, \sec x + \sec x) dx = e^x \sec x + C$

52. When x > 0, then $\int \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) dx$ is

1) $2x \tan^{-1} x - \text{Log}(1 + x^2) + c$ 2) $2x \tan^{-1} x + \text{Log}(1 + x^2) + c$

3) $2[x \tan^{-1} x + \log(1 + x^2)] + c$ 4) $2[x \tan^{-1} x - \log(1 + x^2)] + c$

Ans: (1)

$\int \cos^{-1}\left[\frac{1 - x^2}{1 + x^2}\right] dx = \int \cos^{-1}\left[\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right]$ Put $x = \tan\theta$

$= \int \cos^{-1}(\cos 2\theta)\, dx = \int 2\theta\, dx = 2 \int \tan^{-1} x\, dx = 2\left[\tan^{-1} x . x - \int \frac{1\ x}{1 + x^2} dx\right]$

$= 2\left[x \tan^{-1} x - \frac{1}{2}\log(1 + x^2)\right] + C$

$= 2x \tan^{-1} x - \log(1 + x^2) + C$

53. If the area between $y = mx^2$ and $x = my^2$ $(m > 0)$ is $\frac{1}{4}$ sq units, then value of m is

1) $\sqrt{3}$ 2) $\sqrt{2}$ 3) $\pm 2\sqrt{3}$ 4) $\pm 3\sqrt{2}$

Ans: (answer not found)

$y = mx^2$ $x = my^2$

$x^2 = \frac{1}{m} y$ $y^2 = \frac{1}{m} x$

$4a = m,$ $a = \frac{1}{4} m$

Area $= \frac{16a^2}{3} = \frac{1}{4} \Rightarrow 16 \,.\, \frac{\frac{1}{16m^2}}{3} = \frac{1}{4} \Rightarrow \frac{1}{3m^2} = \frac{1}{4} \Rightarrow m^2 = \frac{4}{3}$

$\Rightarrow m = \pm \frac{2}{\sqrt{3}}$

54. If m and n are degree and order of $(1 + y_1^2)^{2/3} = y_2$, then the value of $\frac{m+n}{m-n}$ is

1) 2 2) 5 3) 4 4) 3

Ans: (2)

$(1 + y_1^2)^{2/3} = y_2$

$(1 + y_1^2)^2 = (y_2)^3$

$\therefore$ m = deg = 3 n = order = 2

$\therefore \frac{m+n}{m-n} = \frac{3+2}{3-2} = 5$

55. The general solution of $\frac{dy}{dx} = 1 - x^2 - y^2 + x^2y^2$ is

1) $2\sin^{-1} y = x\sqrt{1-y^2} + c$ 2) $\sin^{-1} y = \frac{1}{2}\sin^{-1} x + c$

3) $\cos^{-1} y = x \cos^{-1} x + c$ 4) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + c$

Ans: (answer not found)

$\frac{dy}{dx} = 1 - x^2 - y^2 + x^2y^2 = (1 - x^2) - y^2 (1 - x^2)$

$\frac{dy}{dx} = (1 - x^2) (1 - y^2)$

$\frac{dy}{1-y^2} = (1 - x^2)\, dx$

$\int \frac{dy}{1-y^2} = \int (1-x^2)\, dx$

$\frac{1}{2} \log \left(\frac{1+y}{1-y}\right) = x - \frac{x^3}{3} + C$

56. If $x \cos \alpha + y \sin \alpha = 4$ is tangent to $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then the value of α is

1) $\tan^{-1}(3/\sqrt{7})$ 2) $\tan^{-1}(7/3)$ 3) $\tan^{-1}(\sqrt{3}/7)$ 4) $\tan^{-1}(3/7)$

Ans: (1)

$x \cos \alpha + y \sin \alpha = 4$ $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$y \sin \alpha = -x \cos \alpha + 4$

$y = (-\cot \alpha)\, x + 4 \operatorname{cosec} \alpha$

$\therefore m = -\cot \alpha$, $c = 4 \operatorname{cosec} \alpha$, $a^2 = 25$, $b^2 = 9$

$c^2 = a^2m^2 + b^2$

$16\ \text{cosec}^2\alpha = 25\cot^2\alpha + 9$

$16\ (1 + \cot^2\alpha) = 25\cot^2\alpha + 9$

$7 = 9\cot^2\alpha \Rightarrow \cot\alpha = \frac{\sqrt{7}}{3} \Rightarrow \tan\alpha = \frac{3}{\sqrt{7}}\ \therefore\ \alpha = \tan^{-1}(3/\sqrt{7})$

57. If P is a point on $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ with foci S and S', then the maximum value of triangle SPS' is

1) ab/e 2) abe 3) abe^2 4) ab

Ans: (2)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Maximum area = $\frac{1}{2}$. 2ae . b = abe

58. In Argand' plane, the point corresponding to $\frac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)}$ lies in

1) quadrant IV 2) quadrant III 3) quadrant II 4) quadrant I

Ans: (1)

$$\frac{(1-i\sqrt{3})\ (1+i)}{(\sqrt{3}+i)} = \frac{1+i-\sqrt{3}i+\sqrt{3}}{\sqrt{3}+i} = \frac{(1+\sqrt{3})+i(1+\sqrt{3})}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{1}{4}[4-4i] = 1 - i$$

OR

$$\frac{(1-i\sqrt{3})\ (1+i)}{(\sqrt{3}+i)} = \frac{2\,\text{cis}\left(-\frac{\pi}{3}\right).\sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right)}{2\,\text{cis}\left(\frac{\pi}{6}\right)} = \sqrt{2}\ \text{cis}\left(-\frac{\pi}{3}+\frac{\pi}{4}-\frac{\pi}{6}\right) = \sqrt{2}\ \text{cis}\left(-\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right] = 1 - i$$

59. If y = sin x. sin 2x. sin 3x ……… sin nx; then y^{I} is

1) $\sum_{k=1}^{n}\cot kx$ 2) y. $\sum_{k=1}^{n} k\tan kn$ 3) y. $\sum_{k=1}^{n} k\cot kx$ 4) $\sum_{k=1}^{n} k\tan kx$

Ans: (3)

Let n = 1, y = sin x, y' = cos x

Method of inspection

Options (1), (2) and (4) are incorrect

Hence option (4) is correct

60. $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} =$

1) $1 - (\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)(\sin\gamma - \sin\alpha)$

2) $1 + \sin\alpha \sin\beta \sin\gamma$

3) 1

4) 0

Ans: (4)

Let $\alpha = \beta = \gamma = \delta = \frac{\pi}{4}$

$$= \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{vmatrix} = 0$$